

# Annuities

## Practice Problem Set 2

### Future Value of an Annuity

1. On January 1, 2010, you put \$1000 in a savings account that pays  $6\frac{1}{4}\%$  interest, and you will do this every year for the next 18 *[note this correction from the original problem]* years withdraw the balance on December 31, 2028, to pay for your child's college education. How much will you withdraw?

$$\$1000 \times \frac{(1 + .0625)^{18} - 1}{.0625}$$

### Present Value of an Annuity

2. On January 1, 2010, you win a lottery with a payoff of \$2500 at the end of every year for the next 10 years. If you discount the cash flow at 7.8%, what is the least amount you will accept as a single payment right now, instead?

$$\$2500 \times \frac{1 - (1 + .078)^{-10}}{.078}$$

### Future Value of an Annuity Due

3. Suppose in Problem #1, above, you deposited the payments on the last day of each year instead of the first (so you would deposit your last payment on December 31, 2028, then take all the money out the same day)? How much will you withdraw in this case?

$$\$1000 \times \frac{(1 + .0625)^{17} - 1}{.0625} + \$1000$$

### Present Value of an Annuity Due

4. Suppose in Problem #2, above, you receive your first payment right away. What is the minimum amount you would take as a single payoff amount in this case?

$$\$2500 \times \frac{1 - (1 + .078)^{-9}}{.078} + \$2500$$

### Present Value of a Non-Level Annuity

5. On January 1, 2012, you are considering a project that will cost \$3,000,000 right away; it will then pay \$500,000 at the end of each of the first three years; it will then pay \$600,000 in year 4 and \$800,000 in year 5; and then it will return to paying \$500,000 in years 6, 7, and 8. The salvage value at the end of the project life is minimal. If you discount the cash flows for a project with this level of risk at 8.8%, is this project go or no-go?

*You make this an eight-year, level annuity of \$500,000, with a sweetener in year 4 of \$100,000 and another sweetener in year 5 of \$300,000 and find the present value of those three, against the up-front \$3 million cost of the project.*

$$NPV = -3000000 + \$500000 \times \frac{1 - (1 + .088)^{-8}}{.088} + \$100000 \times (1 + .088)^{-4} + \$300000 \times (1 + .088)^{-5}$$

### Future Value of an Annuity with Delay to Withdrawal

6. Suppose in Problem #1 that you do not need the money on December 31, 2028, but that you instead need it 5 years later, on December 21, 2033. You'll stop putting money in the account like you had intended, but you won't pull the money out until the later time. How much money are you going to withdraw at the end of 2033?

*Step 1: Find the future value of the annuity due.*

$$\$1000 \times \frac{(1 + .0625)^{17} - 1}{.0625} + \$1000 = \$29,844.78$$

*Step 2: Take this amount that you will have on December 31, 2028, and let it go forward five years as a lump sum.*

$$\$29,844.78 \times (1 + .0625)^5 = \$40,412.26$$

### Mortgage Payment

7. What would be the monthly payments on a 30-year, 5.89% fixed-rate mortgage for \$200,000? (Assume you borrow the money at the beginning of the month, and your first payment is due at the end of the month.)

*Solve for the payment in the present value of an annuity formula:*

$$\$200,000 = PMT \times \frac{1 - (1 + .0589)^{-30}}{.0589}$$